

Control

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الأساتذة

د. عرفة

محاضرة [3]

$$\text{Ex: } GH(s) = \frac{K}{s(s^2 + 4s + 13)}$$

① Poles $\Rightarrow 0, -2 \pm j3$

Zero $\Rightarrow \phi$

② s-plane

③ Real part

$0 \rightarrow -\infty$

④ Asymptotes

① no. of. asy. = $3 - 0 = 3$

$$\textcircled{2} \sigma_c = \frac{0 - 2 - j3 - 2 + j3 - 0}{3}$$

$$= \frac{-4}{3} = -1.33$$

$$\theta = \frac{(2L + 1)180}{3}$$

$$\theta_1 = 60^\circ, \theta_2 = 180^\circ, \theta_3 = 300^\circ = -60^\circ$$

نقطة ال Breaking point غير ضرورية حيث أنه لا توجد

⑤ departure angle

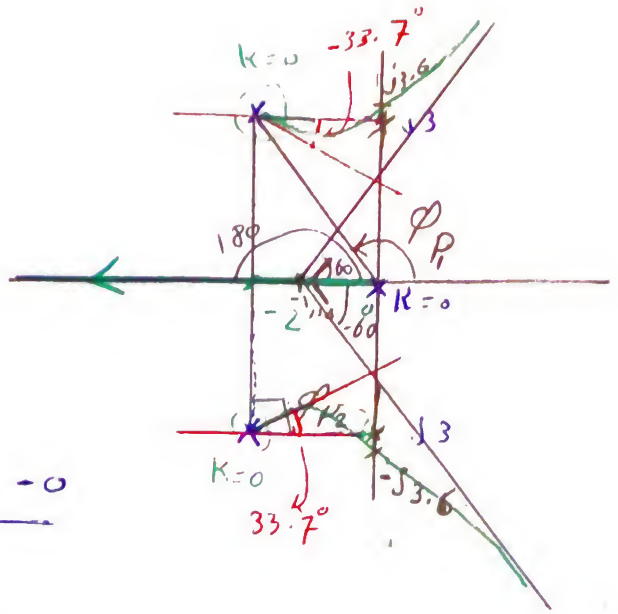
$$\text{departure } \angle_1 = 180 - \phi_p + \phi_z$$

$$\theta_{D_{-2+j3}} = 180 - (\phi_{p_1} + \phi_{p_2}) + 0$$

$$= 180 - \left[(180 - \tan^{-1} \frac{3}{2}) + 90^\circ \right] + 0$$

$$= -33.7^\circ$$

$$\theta_{D_{-2-j3}} = 33.7^\circ$$



⑥ Range of stability for K

At imag Axis

$$\text{ch. eq.} \Rightarrow 1 + G H(s) = 0$$

$$1 + \frac{K}{s^3 + 4s^2 + 13s} = 0$$

$$s^3 + 4s^2 + 13s + K = 0$$

$$0 < K_1 < \textcircled{52} \rightarrow K_c$$

Aux eq A(s)

$$A(s) = 4s^2 + K$$

$$4s^2 + K_c = 0$$

$$4s^2 + 52 = 0 \Rightarrow s_{1,2} = \left(\pm j 3.6 \right)$$

تقاطع المحاور الرأسية root locus مع المحاور الرأسية

| | | |
|-------|--------------------------------------|----|
| s^3 | 1 | 13 |
| s^2 | 4 | K |
| s^1 | $\frac{52-K}{4} > 0 \textcircled{1}$ | |
| s^0 | $K > 0 \textcircled{2}$ | |

for ①

$$52 - K > 0$$

$$K < 52$$

range of
stability

$$\text{Ex: } G H(s) = \frac{K(s+2)}{s^2 + 4s + 20}$$

$$\textcircled{1} \text{ Poles} \Rightarrow -2 + j4, -2 - j4$$

$$\textcircled{2} \text{ Zeros} \Rightarrow -2$$

② s-plane \Rightarrow turn over

③ Real Part

$$-2 \rightarrow -\infty$$

④ Breaking Point

ch. eq. $1 + GH(s) = 0$

$$GH(s) = -1$$

$$K = - \frac{s^2 + 4s + 20}{s + 2}$$

$$\frac{dK}{ds} = 0 \Rightarrow - \frac{(2s + 4)(s + 2) - (s^2 + 4s + 20)}{(s + 2)^2} = 0$$

$$\therefore (s + 2)(2s + 4) - (s^2 + 4s + 20) = 0$$

$$2s^2 + 8s + 8 - s^2 - 4s - 20 = 0$$

$$s^2 + 4s - 12 = 0$$

$$(s + 6)(s - 2) \Rightarrow s_{1,2} = 2, -6$$

Accepted

Breaking point (s_b)

$$s_b = -6 \Rightarrow K|_{s_b = -6} = \frac{(-6)^2 - 4(-6) + 20}{-6 + 2} = 8$$

⑤ departure angle

$$\theta_D = 180 - \phi_P + \phi_Z$$

$$\phi_D = 180 - 90 + 90 = 180^\circ$$

$$\phi_D = -180^\circ$$

Stable for $K > 0$

$$\text{Ex: } GH(s) = \frac{20(1+KS)}{s(s+1)(s+4)}$$

ch. eq:

$$1 + GH(s) = 0$$

$$1 + \frac{20 + 20KS}{s(s+1)(s+4)}$$

$$s(s+1)(s+4) + 20 + 20KS = 0$$

$$1 + \frac{20KS}{s(s+1)(s+4) + 20} = 0$$

$\rightarrow GH(s)$

$$GH(s) = \frac{20KS}{s(s+1)(s+4) + 20}$$

Poles $\Rightarrow -5, j2, -j2$

Zero $\Rightarrow 0$

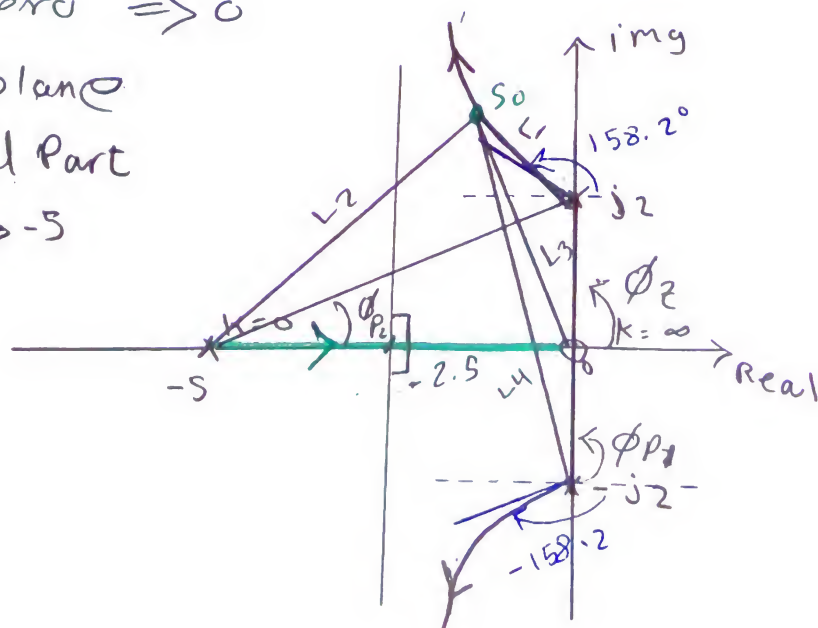
① Poles $\Rightarrow -5, j2, -j2$

Zero $\Rightarrow 0$

② s-plane

③ Real Part

$0 \rightarrow -5$



\Rightarrow Turn over

④ Asy. → ① No. of asy. $-3-1=2$

→ ② $C_A = \frac{(-5+j2-j'2)-6}{2}$
 $= \frac{-5}{2} = -2.5$

→ ③ $\theta = \frac{(2L+1)180}{2}$

$$\theta_1 = 90$$

$$\theta_2 = -90$$

⑤ departure Angle:

$$\begin{aligned}\theta_{D_{j_2}} &= 180 - (\phi_{P_1} + \phi_{P_2}) + \phi_z \\ &= 180 - (90 + \tan^{-1}(\frac{2}{5})) + 90^\circ \\ &= 158.2^\circ\end{aligned}$$

$$\theta_{D_{-j_2}} = -158.2^\circ$$

System is stable for
 $K > 0$

① Find K at S_0 .

$$K \Big|_{S_0} = \frac{\prod \text{Poles}}{\prod \text{Zero}} = \frac{L_1 \cdot L_2 \cdot L_3}{L_4}$$

② find K at $\zeta = 0.5$

$$T.F. = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$S_{1,2} = -\zeta\omega_n \pm j\omega_d$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

② K at $\zeta = 0.5$

$$\phi = \cos^{-1} \zeta = 60^\circ$$

③ K at $t_s = \sqrt{\text{sec}}$

given $\zeta \omega_n$

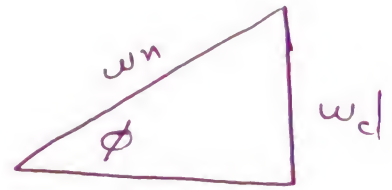
$$t_s = \frac{4}{\zeta \omega_n} \text{ (2% error)}$$

دستگاه را می‌خواهیم در 2٪ خطا قرار دهیم

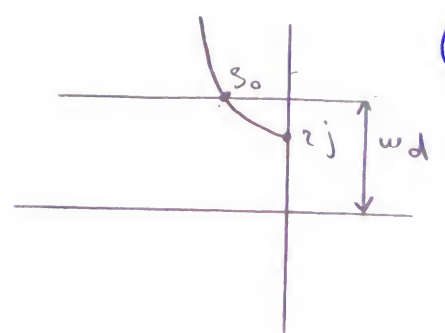
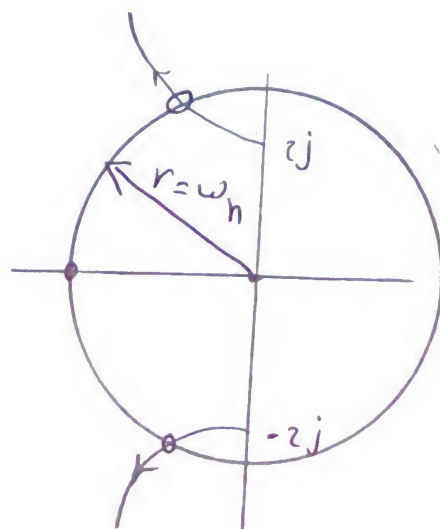
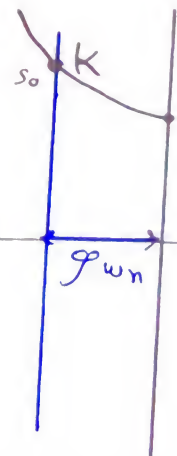
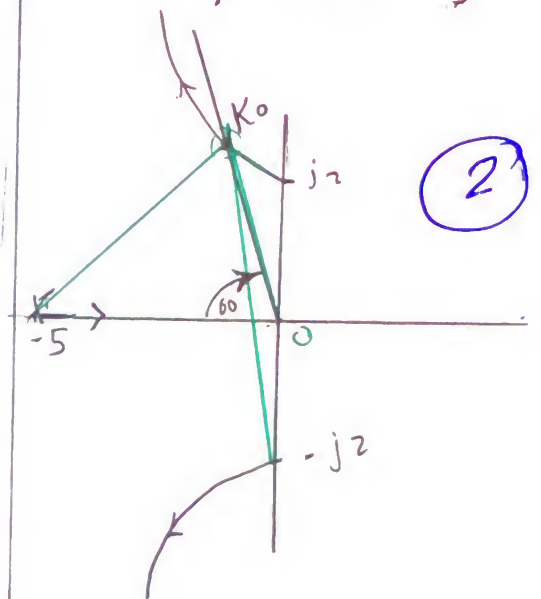
در این حالت $\zeta \omega_n$

④ K at $\omega_d = \sqrt{\text{rad/sec}}$
damp. freq. \rightarrow

⑤ Find K $\omega_n = \text{--- rad/sec}$



$$\phi = \cos^{-1} \zeta$$



Report:

$$GH(s) = \frac{K(s+4)(s+6)}{s(s+2)}$$

- Draw the root locus

- Find the range of K for stability

- K at $t_s = 2 \text{ sec} = \frac{4}{\zeta \omega_n} \Rightarrow \zeta \omega_n = 2$

+ K at $\varphi = \frac{\sqrt{3}}{2} \Rightarrow \phi = \cos^{-1} \frac{\sqrt{3}}{2} = 30^\circ$

